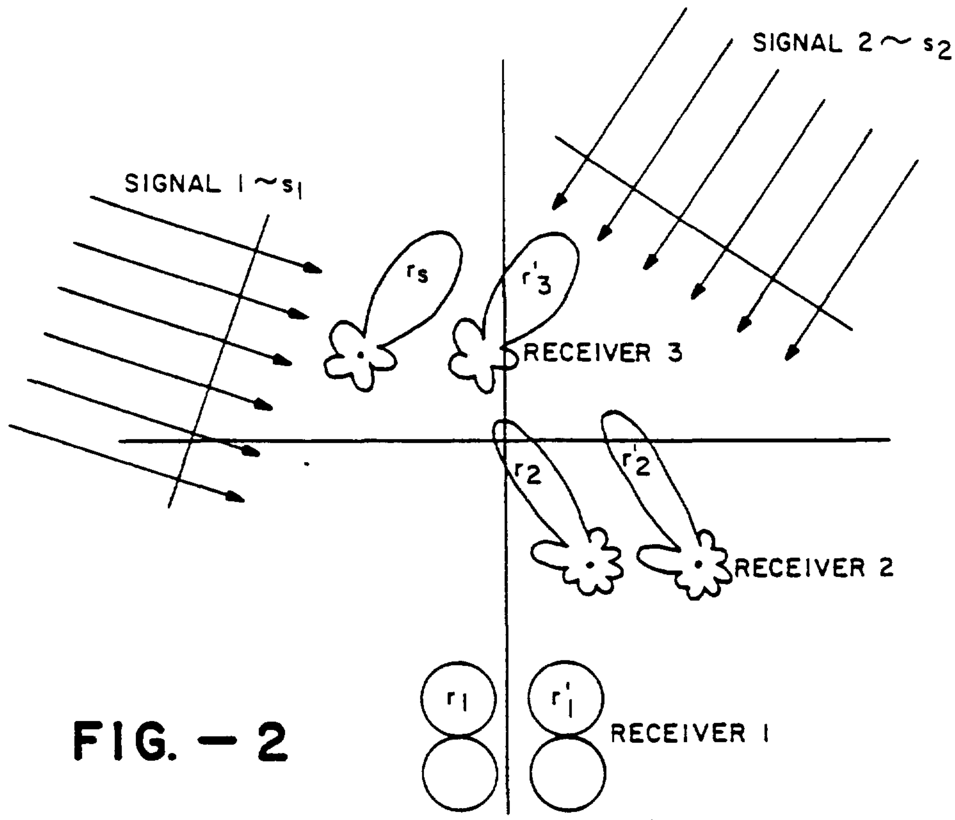
SIGNAL 2 ~ s_2
SIGNAL 1 ~ s_1
r_s
r'_3
RECEIVER 3
r_2
r'_2
RECEIVER 2
r_1
r'_1
RECEIVER 1

FIG. - 2

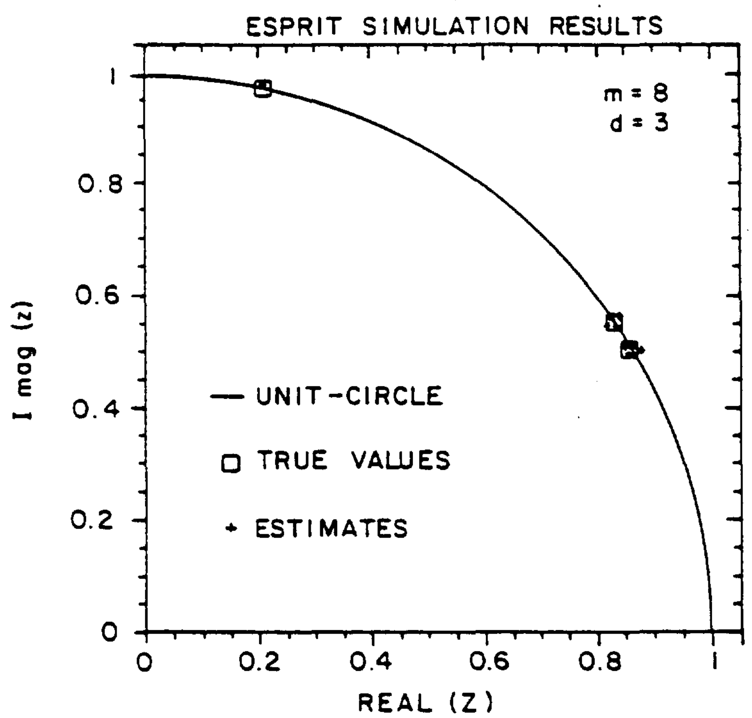
ESPRIT SIMULATION RESULTS
m = 8
d = 3
I mag (z)
1
0.8
0.6
0.4
0.2
0
UNIT-CIRCLE
TRUE VALUES
ESTIMATES
0
0.2
0.4
0.6
0.8
1
REAL (Z)

FIG.-3

METHOD FOR ESTIMATING SIGNAL SOURCE LOCATIONS AND SIGNAL PARAMETERS USING AN ARRAY OF SIGNAL SENSOR PAIRS

The U.S. Government has rights in the described and claimed invention pursuant to Department of Navy Contract N00014-85-K-0550 and Department of Army Agreement No. DAAG29-85-K-0048.

BACKGROUND OF THE INVENTION

The invention described in this patent application relates to the problem of estimation of constant parameters of multiple signals received by an array of sensors in the presence of additive noise. There are many physical problems of this type including direction finding (DF) wherein the signal parameters of interest are the directions-of-arrival (DOA's) of wavefronts impinging on an antenna array (cf. FIG. 1), and harmonic analysis in which the parameters of interest are the temporal frequencies of sinusoids contained in a signal (waveform) which is known to be composed of a sum of multiple sinusoids and possibly additive measurement noise. In most situations, the signals are characterized by several unknown parameters all of which need to be estimated simultaneously (e.g., azimuthal angle, elevation angle and temporal frequency) and this leads to a multidimensional parameter estimation problem.

High resolution parameter estimation is important in many applications including electromagnetic and acoustic sensor systems (e.g., radar, sonar, electronic surveillance systems, and radio astronomy), vibration analysis, medical imaging, geophysics, well-logging, etc.. In such applications, accurate estimates of the parameters of interest are required with a minimum of computation and storage requirements. The value of any technique for obtaining parameter estimates is strongly dependent upon the accuracy of the estimates. The invention described herein yields accurate estimates while overcoming the practical difficulties encountered by present methods such as the need for detailed a priori knowledge of the sensor array geometry and element characteristics. The technique also yields a dramatic decrease in the computational and storage requirements.

The history of estimation of signal parameters can be traced back at least two centuries to Gaspard Riche, Baron de Prony, (R. Prony, Essai experimental et analytic, etc. L'Ecole Polytechnique, 1: 24–76, 1795) who was interested in fitting multiple sinuisoids (exponentials) to data. Interest in the problem increased rapidly after World War II due to its applications to the fast emerging technologies of radar, sonar and seismology. Over the years, numerous papers and books addressing this subject have been published, especially in the context of direction finding in passive sensor arrays.

One of the earliest approaches to the problem of direction finding is now commonly referred to as the conventional beamforming technique. It uses a type of matched filtering to generate spectral plots whose peaks provide the parameter estimates. In the presence of multiple sources, conventional beamforming can lead to signal suppression, poor resolution, and biased parameter (DOA) estimates.

The first high resolution method to improve upon conventional beamforming was presented by Burg (J. P. Burg, Maximum entropy spectral analysis, In *Proceedings of the 37th Annual International SEG Meeting*, Oklahoma City, OK., 1967). He proposed to extrapolate the array covariance function beyond the few measured bags, selecting that extrapolation for which the entropy of the signal is maximized. The Burg technique gives good resolution but suffers from parameter bias and the phenomenon referred to as line splitting wherein a single source manifests itself as a pair of closely spaced peaks in the spectrum. These problems are attributable to the mismodeling inherent in this method.

A different approach aimed at providing increased parameter resolution was introduced by Capon (J. Capon, High resolution frequency wave number spectrum analysis, *Proc. IEEE*, 57: 1408–1418, 1969). His approach was to find a weight vector for combining the outputs of all the sensor elements that minimizes output power for each look direction while maintaining a unit response to signals arriving from this direction. Capon's method has difficulty in multipath environments and offers only limited improvements in resolution.

A new genre of methods were introduced by Pisarenko (V. F. Pisarenko, The retrieval of harmonics from a covariance function, *Geophys. J. Royal Astronomical Soc.*, 33: 347–366, 1973) for a somewhat restricted formulation of the problem. These methods exploit the eigenstructure of the array covariance matrix. Schmidt made important generalizations of Pisarenko's ideas to arbitrary array/wavefront geometries and source correlations in his Ph.D. thesis titled *A Signal Subspace Approach to Multiple Emitter Location and Spectral Estimation*, Stanford University, 1981. Schmidt's MUltiple SIgnal Classification (MUSIC) algorithm correctly modeled the underlying problem and therefore generated superior estimates. In the ideal situation where measurement noise is absent (or equivalently when an infinite amount of measurements are available), MUSIC yields exact estimates of the parameters and also offers infinite resolution in that multiple signals can be resolved regardless of the proximity of the signal parameters. In the presence of noise and where only a finite number of measurements are available, MUSIC estimates are very nearly unbiased and efficient, and can resolve closely spaced signal parameters.

The MUSIC algorithm, often referred to as the eigenstructure approach, is currently the most promising high resolution parameter estimation method. However, MUSIC and the earlier methods of Burg and Capon which are applicable to arbitrary sensor array configurations suffer from certain shortcomings that have restricted their applicability in several problems. Some of these are:

Array Geometry and Calibration—A complete characterization of the array in terms of the sensor geometry and element characteristics is required. In practice, for complex arrays, this characterization is obtained by a series of experiments known as array calibration to determine the so called array manifold. The cost of array calibration can be quite high and the procedure is sometimes impractical. Also, the associated storage required for the array manifold is $2ml^g$ words (m is the number of sensors, l is the number of search (grid) points in each dimension, and g is the number of dimensions) and can become large even for simple applications. For example, a sensor array containing 20 elements, searching over a hemisphere with a 1 millirad resolution in azimuth and elevation and using 16 bit words (2 bytes each) requires approximately 100 megabytes of storage! This number increases exponentially as another search dimension such as temporal frequency is included. Furthermore, in certain applications the array geometry may be slowly changing such as in light weight spaceborne antenna structures, sonobuoy and towed arrays used in sonar etc., and a complete characterization of the array is never available.

Computational Load—In the prior methods of Burg, Capon, Schmidt and others, the main computational burden lies in generating a spectral plot whose peaks correspond to the parameter estimates. For example, the number of operations required in the MUSIC algorithm in order to compute the entire spectrum, is approximately $4m^2l^{\nu}$. An operation is herein considered to be a floating point multiplication and an addition. In the example above, the number of operations needed is approximately 4×10^9 which is prohibitive for most applications. A powerful 10 MIP (10 million floating point instructions per second) machine requires about 7 minutes to perform these computations! Moreover, the computation requirement grows exponentially with dimension of the parameter vector. Augmenting the dimension of the parameter vector further would make such problems completely intractable.

The technique described herein is hereafter referred to as Estimation of Signal Parameters using Rotational Invariance Techniques (ESPRIT). ESPRIT obviates the need for array calibration and dramatically reduces the computational requirements of previous approaches. Furthermore, since the array manifold is not required, the storage requirements are eliminated altogether.

SUMMARY OF THE INVENTION

ESPRIT is an alternative method for signal reception and source parameter estimation which possesses most of the desirable features of prior high resolution techniques while realizing substantial reduction in computation and elimination of storage requirements. The basic properties of the invention may be summarized as follows:

1. ESPRIT details a new method of signal reception for source parameter estimation for planar wavefronts.
2. The method yields signal parameter estimates without requiring knowledge of the array geometry and sensor element characteristics, thus eliminating the need for sensor array calibration.
3. ESPRIT provides substantial reduction in computation and elimination of storage requirements over prior techniques. Referring to the previous example, ESPRIT requires only 4×10^4 computations compared to 4×10^9 computations required by prior methods, and reduces the time required from 7 minutes to under 4 milliseconds. Furthermore, the 100 megabytes of storage required is completely eliminated.
4. A feature of the invention is the use of an array of sensor pairs where the sensors in each pair are identical and groups of pairs have a common displacement vector.

Briefly, in accordance with the invention, an array of signal sensor pairs is provided in which groups of sensor pairs have a uniform relative vector displacement within each group, but the displacement vector for each group is unique. The sensors in each pair must be matched, however they can differ from other sensor pairs. Moreover, the characteristics of each sensor and the array geometry can be arbitrary and need not to be known. Within each group, the sensor pairs can be arranged into two subarrays, X and Y, which are identical except for a fixed displacement (cf. FIG. 2). For example, in order to simultaneously perform temporal frequency and spatial angle estimation, one group of sensor pairs would share a common spatial displacement vector while the second group would share a common temporal displacement. In general, for each additional type of parameter to be estimated, a sensor group sharing a common displacement is provided. Furthermore, the number of sensor pairs in each group must be more than the number of sources whose parameters are to be estimated.

Having provided an array of sensors which meets the specifications outlined above, signals from this array of sensor pairs are then processed in order to obtain the parameter estimates of interest. The procedure for obtaining the parameter estimates may be outlined as follows:

1. Using the array measurements from a group of sensor pairs, determine the auto-covariance matrix R_{xx} of the X subarray in the group and the cross-covariance matrix R_{xy} between the X and Y subarrays in the group.
2. Determine the smallest eigenvalue of the covariance matrix R_{xx} and then subtract it out from each of the elements on the principal diagonal of R_{xx}. The results of the subtraction are referred to hereinafter as C_{xx}.
3. Next, the generalized eigenvalues (GE's) γ_i of the matrix pair (C_{xx}, R_{xy}) are determined. A number d of the GE's will lie on or near the unit circle and the remaining $m-d$ noise GE's will lie at or near the origin. The number of GE's on or near the unit circle determine the number of sources, and their angles are the phase differences sensed by the sensor doublets in the group for each of the wavefronts impinging on the array. These phase differences are directly related to the directions of arrival.
4. The procedure is then repeated for each of the groups, thereby obtaining the estimates for all the parameters of interest (e.g., azimuth, elevation, temporal frequency).

Thus, the number of sources and the parameters of each source are the primary quantities determined. ESPRIT can be further extended to the problem of determining the array geometry a posteriori, i.e., obtaining estimates of the sensor locations given the measurements. Source powers and optimum weight vectors for solving the signal copy problem, a problem involving estimation of the signal received from one of the sources at a time eliminating all others, can also be estimated in a straightforward manner as follows:

1. The optimum weight vector for signal copy for the i^{th} signal is the generalized eigenvector (GV) e_i corresponding to the i^{th} GE γ_i.
2. For the case when the sources are uncorrelated, the direction vector a_i for the i^{th} wavefront is given by $R_{xy}e_i$. With these direction vectors in hand, the array geometry can be estimated by solving a set of linear equations.
3. Using the direction vectors a_i, the signal powers can also be estimated by solving a set of linear equations.

The invention and objects and features thereof will be more readily apparent from the following example and appended claims.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. 1 is a graphic representation of a problem of direction-of-arrival estimation in which two sources are present and being monitored by a three-element array of sensors.

FIG. 2 is a graphic representation of a similar problem in which the two signals are now impinging on an array of sensors pairs in accordance with the invention.

FIG. 3 is a graphic illustration of the parameter estimates from a simulation performed in accordance with the invention in which three signals were impinging on an array of eight sensor doublets and directions-of-arrival were being estimated.

DETAILED DESCRIPTION OF THE DRAWINGS

As indicated above, the invention is directed at the estimation of constant parameters of signals received by an array of sensor pairs in the presence of noise. The problem can be visualized with reference to FIG. 1 in which two signals (s_1 and s_2) are impinging on an array of three sensors (r_1, r_2, r_3). It is assumed in this illustrated example that the sources and sensors lie in a plane; thus only two parameters need be identified, the azimuth angle of the two signals. Heretofore, techniques such as MUSIC have been able to accurately estimate the DOA's of the two signals; however the characteristics of each sensor must be known as well as the overall array geometry. This leads to exceedingly large storage requirements when the array must be calibrated, and a correspondingly large computation time in the execution of the algorithms.

In accordance with the present invention, array (manifold) calibration is not required in ESPRIT as long as the array is comprised of (groups of) matched sensor pairs sharing a common displacement vector. This is illustrated in FIG. 2 in which the two signals (s_1 and s_2) are sensed by receiver pairs (r_1, r'_1; r_2, r'_2; and r_3, r'_3). The only requirements of the array are that the sensor pairs are offset by the same vector as indicated, and that the number of sensor pairs exceeds the number of sources as is the case in this example.

The performance of the invention is graphically illustrated in FIG. 3 which presents the results of a simulation performed according to the specifications of ESPRIT. The simulation consisted of an array with 8 doublets. The elements in each of the doublets were spaced a quarter of a wavelength apart. The array geometry was generated by randomly scattering the doublets on a line 10 wavelengths in length such that the doublet axes were all parallel to the line. Three planar and weakly correlated signal wavefronts impinged on the array at angles 20°, 22°, and 60°, with SNRs of 10, 13 and 16 db relative to the additive uncorrelated noise present at the sensors. The covariance estimates were computed from 100 snapshots of data and several simulations runs were made using independent data sets.

FIG. 3 shows a plot of the GE's obtained from 10 independent trials. The three small circles on the unit circle indicate the locations of the true parameters and the pluses are the estimates obtained using ESPRIT. The GE's on the unit circle are closely clustered and the two sources 2° apart are easily resolved.

As illustrated, accurate estimates of the DOA's are obtained. Furthermore, ESPRIT has several additional features which are enumerated below.

1. ESPRIT appears to be very robust to errors in estimating the minimum eigenvalue of the covariance R_{xx}. It is also robust to the numerical properties of the algorithm used to estimate the generalized eigenvalues.
2. ESPRIT does not require the estimation of the number of sources prior to source parameter estimation as in the MUSIC algorithm, where an error in the estimate of the number of sources can invalidate the parameter estimates. In accordance with the invention, ESPRIT simultaneously estimates the signal parameters and the number of sources.

APPLICATIONS

There are a number of applications that exploit one or more of the important features of ESPRIT, i.e., its insensitivity to array geometry, low computational load and no storage requirements. Some of these are described below.

1. Direction-of-Arrival Estimation

(a) Space Antennas—Space structures are necessarily light weight, very large and therefore fairly flexible. Small disturbances can cause the structure to oscillate for long periods of time resulting in a sensor array geometry which is time-varying. Furthermore, it is nearly impossible to completely calibrate such an array as the setting up of a suitable facility is not practical. On the other hand, the use of matched pairs of sensor doublets whose directions are constantly aligned by a low-cost star-tracking servo results in total insensitivity to the global geometry of the array. Note that signal copy can still be performed, a function which, is often a main objective of such large spaceborne antenna arrays. In fact, a connected structure for the array is not required! Rather, only a collection of relatively small antenna doublets is needed, each possessing a star-tracker or earth-based beacon tracker for alignment. Ease of deployment, maintenance, and repair of such disconnected arrays can have significant cost and operational benefits (for example, a defective unit can be merely transported to a space station or back to the earth for repair).

(b) Sonobuoys—Sonobuoys are air-dropped and scatter somewhat randomly on the ocean surface. The current methods of source location require complete knowledge of the three dimensional geometry of the deployed array. The determination of the array geometry is both expensive and undesirable (since it involves active transmission thus alerting unfriendly elements!). Using ESPRIT, vertical alignment of doublets can be achieved using gravity as a reference. Horizontal alignment can be obtained via a small servo and a miniature magnetic sensor (or even use an acoustic spectral line radiated from a beacon or the target itself). Within a few minutes after the sonobuoys are dropped, alignment can be completed and accurate estimates of DOA's become available. As before, signal copy processing is also feasible. Furthermore, the sonobuoy array geometry can itself be determined should this be of interest.

(c) Towed Arrays—These consist of a set of hydrophones placed inside a acoustically transparent tube that is towed well behind a ship or submarine. The common problem with towed arrays is that the tube often distorts from the assumed straight line geometry due to ocean and tow-ship induced disturbances. Therefore, prior array calibration becomes invalid. In the new approach, any translational disturbance in the doublets is of no consequence. Therefore by selective use of doublets (whose orientation can be easily sensed) that are acceptably co-directional, reliable source DOA estimates can still be obtained.

(d) Mobile DF and Signal Copy Applications—Often, mobile (aircraft, van mounted) direction finding (DF) systems cannot meet the vast storage and computational requirements of the prior methods. ESPRIT can drastically reduce such requirements and still provide good performance. This has particular applicability in the field of cellular mobile communications where the number of simultaneous users is limited due to finite bandwidth constraints and cross-talk (inter-channel interference). Current techniques for increasing the number of simultaneous users exploit methods of signal separation such as frequency, time and code division multiplexing apart from the area multiplexing inherent to the cellular concept. Using directional discrimination (angle division multiplexing), the number of simultaneous users could be increased significantly. ESPRIT provides a simple and relatively low cost technique for performing the signal copy operation through angular signal separation. The estimation (possibly recursively) of the appropriate generalized eigenvector is all that is needed in contrast to substantially more complex procedures required by prior methods.

2. Temporal Frequency Estimation—There are many applications in radio astronomy, modal identification of linear systems including structural analysis, geophysics sonar, electronic surveillance systems, analytical chemistry etc., where a composite signal containing multiple harmonics is present in additive noise. ESPRIT provides frequency estimates from suitably sampled time series at a substantially reduced level of computation over the previous methods.
3. Joint DOA-Frequency Estimation—Applications such as radio astonomy may require the estimation of declination and right ascension of radio sources along with the frequency of the molecular spectral lines emitted by them. Such problems also arise in passive sonar and electronic surveillance applications. As previously noted, ESPRIT has particularly important advantages in such multi-dimensional estimation problems.

Having concluded the summary of the invention and applications, a detailed mathematical description of the invention is presented.

PROBLEM FORMULATION

The basic problem under consideration is that of estimation of parameters of finite dimensional signal processes given measurements from an array of sensors. This general problem appears in many different fields including radio astronomy, geophysics, sonor signal processing, electronic surveillance, structural (vibration) analysis, temporal frequency estimation, etc. In order to simplify the description of the basic ideas behind ESPRIT, the ensuing discussion is couched in terms of the problem of multiple source direction-of-arrival (DOA) estimation from data collected by an array of sensors. Though easily generalized to higher dimensional parameter spaces, the discussion and results presented deal only with single dimensional parameter spaces, i.e., azimuth only direction finding (DF) of far-field point sources. Furthermore, narrowband signals of known center frequency will be assumed. A DOA/DF problem is classified as narrowband width is small compared to the inverse of the transit time of a wavefront across the array. The generality of the fundamental concepts on which ESPRIT is based makes the extension to signals containing multiple frequencies straightforward as discussed later. Note that wideband signals can also be handled by decomposing them into narrowband signal sets using comb filters.

Consider a planar array of arbitrary geometry composed of m matched sensor doublets whose elements are translationally separated by a known constant displacement vector as shown in FIG. 2. The element characteristics such as element gain and phase pattern, polarization sensitivity, etc., may be arbitrary for each doublet as long as the elements are pairwise identical. Assume there are $d<m$ narrowband stationary zero-mean sources centered at frequency ω_0, and located sufficiently far from the array such that in homogenous isotropic transmission media, the wavefronts impinging on the array are planar. Additive noise is present at all the $2m$ sensors and is assumed to be a stationary zero-mean random process that is uncorrelated from sensor to sensor.

In order to exploit the translational invariance property of the sensor array, it is convenient to describe the array as being comprised of two subarrays, X and Y, identical in every respect although physically displaced (not rotated) from each other by a known displacement vector. The signals received at the i^{th} doublet can then be expressed as:

$$x_i(t) = \sum_{k=1}^{d} s_k(t)\, a_i(\theta_k) + n_{x_i}(t) \tag{1}$$

$$y_i(t) = \sum_{k=1}^{d} s_k(t)\, e^{j\omega_0 \Delta \sin\theta_k/c}\, a_i(\theta_k) + n_{y_i}(t)$$

where $s_k(\cdot)$ is the k^{th} signal (wavefront) as received at sensor 1 (the reference sensor) of the X subarray, θ_k is the direction of arrival of the k^{th} source relative to the direction of the translational displacement vector, $a_i(\theta_k)$ is the response of the i^{th} sensor of either subarray relative to its response at sensor 1 of the same subarray when a single wavefront impinges at an angle θ_k, Δ is the magnitude of the displacement vector between the two arrays, c is the speed of propagation in the transmission medium, $n_{x_i}(\cdot)$ and $n_{y_i}(\cdot)$ are the additive noises at the elements in the i^{th} doublet for subarrays X and Y respectively.

Combining the outputs of each of the sensors in the two subarrays, the received data vectors can be written as follows:

$$x(t) = As(t) + n_x(t),$$

$$y(t) = A\Phi s(t) + n_y(t); \tag{2}$$

where:

$$x^T(t) = [x_1(t) \ldots x_m(t)],$$

$$n_x^T(t) = [n_{x1}(t) \ldots n_{xm}(t)],$$

$$y^T(t) = [y_1(t) \ldots y_m(t)],$$

$$n_y^T(t) = [n_{y1}(t) \ldots n_{ym}(t)], \tag{3}$$

The vector s(t) is a $d \times 1$ vector of impinging signals (wavefronts) as observed at the reference sensor of subarray X. The matrix Φ is a diagonal $d \times d$ matrix of the phase delays between the doublet sensors for the d wavefronts, and can be written as:

$$\Phi = \mathrm{diag}[e^{j\omega_0\Delta \sin\theta_1/c}, \ldots, e^{j\omega_0\Delta \sin\theta_d/c}]. \tag{4}$$

Note that Φ is a unitary matrix (operator) that relates the measurements from subarray X to those from subarray Y. In the complex field, Φ is a simple scaling operator. However, it is isomorphic to the real two-dimensional rotation operator and is herein referred to as a rotation operator. The $m \times d$ matrix A is the direction matrix whose columns $\{a(\theta_k), k=1, \ldots, d\}$ are the signal direction vectors for the d wavefronts.

$$a^T(\theta_k)=[a_1(\theta_k), \ldots, a_m(\theta_k)]. \tag{5}$$

The auto-covariance of the data received by subarray X is given by:

$$R_{xx}=E[x(t)x^*(t)]=ASA^*+\sigma^2 I, \tag{6}$$

where S is the $d \times d$ covariance matrix of the signals $s(t)$, i.e.,

$$S=E[s(t)s(t)^*], \tag{7}$$

and σ^2 is the covariance of the additive uncorrelated white noise that is present at all sensors. Note that $(\cdot)^*$ is used herein to denote the Hermitean conjugate, or complex conjugate transpose operation. Similarly, the cross-covariance between measurements from subarrays X and Y is given by:

$$R_{xy}=E[x(t)y(t)^*]=AS\Phi^*A^*. \tag{8}$$

This completes the definition of the signal and noise model, and the problem can now be stated as follows:

Given measurements $x(t)$ and $y(t)$, and making no assumptions about the array geometry, element characteristics, DOA's, noise powers, or the signal (wavefront) correlation, estimate the signal DOA's.

ROTATIONALLY INVARIANT SUBSPACE APPROACH

The basic idea behind the new technique is to exploit the rotational invariance of the underlying signal subspaces induced by the translational invariance of the sensor array. The following theorem provides the foundation for the results presented herein.

Theorem: Define Γ as the generalized eigenvalue matrix associated with the matrix pencil $\{(R_{xx}-\lambda_{min}I), R_{xy}\}$ where λ_{min} is the minimum (repeated) eigenvalue of R_{xx}. Then, if S is nonsingular, the matrices Φ and Γ are related by

$$\Gamma=\begin{bmatrix}\Phi & 0\\ 0 & 0\end{bmatrix} \tag{9}$$

to within a permutation of the elements of Φ.

Proof: First it is shown that ASA^* is rank d and R_{xx} has a multiplicity $(m-d)$ of eigenvalues all equal to σ^2. From linear algebra,

$$\rho(ASA^*)=\min(\rho(A),\rho(S)) \tag{10}$$

where $\rho(\cdot)$ denotes the rank of the matrix argument. Assuming that the array geometry is such that there are no ambiguities (at least over the angular interval where signals are expected), the columns of the $m \times d$ matrix A are linearly independent and hence $\rho(A)=d$. Also, since S is a $d \times d$ matrix and is nonsingular, $\rho(S)=d$. Therefore, $\rho(ASA^*)=d$, and consequently ASA^* will have $m-d$ zero eigenvalues. Equivalently $ASA^*+\sigma^2 I$ will have $m-d$ minimum eigenvalues all equal to σ^2. If $\{\lambda_1>\lambda_2> \ldots >\lambda_m\}$ are the ordered eigenvalues of R_{xx}, then

$$\lambda_{d+1}= \ldots =\lambda_m=\sigma^2. \tag{11}$$

Hence,

$$R_{xx}-\lambda_{min}I=R_{xx}-\sigma^2 I=ASA^*. \tag{12}$$

Now consider the matrix pencil

$$C_{xx}-\gamma R_{xy}=ASA^*-\gamma AS\Phi^*A^*=AS(I-\gamma\Phi^*)A^*; \tag{13}$$

where $C_{xx} \doteq R_{xx}-\lambda_{min}^{xx}I$. By inspection, the column space of both ASA^* and $AS\Phi^*A^*$ are identical. Therefore, $\rho(ASA^*-\gamma AS\Phi^*A^*)$ will in general be equal to d. However, if

$$\gamma=e^{j\omega_0\Delta \sin\theta_i/c}, \tag{14}$$

the i^{th} row of $(I-e^{j\omega_0\Delta \sin\theta_i/c}\Phi)$ will become zero. Thus,

$$\rho(I-e^{j\omega_0\Delta \sin\theta_i/c}\Phi)=d-1. \tag{15}$$

Consequently, the pencil $(C_{xx}-\gamma R_{xy})$ will also decrease in rank to $d-1$ whenever γ assumes values given by (14). However, by definition these are exactly the generalized eigenvalues (GEV's) of the matrix pair $\{C_{xx}, R_{xy}\}$. Also, since both matrices in the pair span the same subspace, the GEV's corresponding to the common null space of the two matrices will be zero, i.e., d GEV's lie on the unit circle and are equal to the diagonal elements of the rotation matrix Φ, and the remaining $m-d$ (equal to the dimension of the common null space) GEV's are at the origin. This completes the proof of the theorem.

Once Φ is known, the DOA's can be calculated from:

$$\theta_k=\arcsin\{c\Phi_k k/\omega_0\Delta\}. \tag{16}$$

Due to errors in estimating R_{xx} and R_{xy} from finite data as well as errors introduced during the subsequent finite precision computations, the relations in (9) and (11) will not be exactly satisfied. At this point, a procedure is proposed which is not globally optimal, but utilizes some well established, stepwise-optimal techniques to deal with such issues.

Subspace Rotation Algorithm (ESPRIT)

The key steps of the algorithm are:

1. Find the auto- and cross-covariance matrix estimates $\hat{R}_{xx}$ and $\hat{R}_{xy}$ from the data.
2. Compute the eigen-decomposition of $\hat{R}_{xx}$ and $\hat{R}_{xy}$ and then estimate the number of sources $\hat{d}$ and the noise variance $\hat{\sigma}^2$.
3. Compute rank $\hat{d}$ approximations to ASA^* and $AS\Phi^*A^*$ given $\hat{\sigma}^2$.
4. The $\hat{d}$ GEV's of the estimates of ASA^* and $AS\Phi^*A^*$ that lie close to the unit circle determine the subspace rotation operator Φ and hence, the DOA's.

Details of the algorithm are now discussed.

Covariance Estimation

In order to estimate the required covariances, observations $x(t_j)$ and $y(t_j)$ at time intervals t_j are required. Note that the subarrays must be sampled simultaneously. The maximum likelihood estimates (assuming no underlying data model) of the auto- and cross-covariance matrices are then given by

$$\hat{R}_{xx} = \frac{1}{N}\sum_{j=1}^{N} x(t_j)x(t_j)^* \tag{17}$$

$$\hat{R}_{xy} = \frac{1}{N}\sum_{j=1}^{N} x(t_j)y(t_j)^*.$$

The number of snapshots, N, needed for an adequate estimate of the covariance matrices depends upon the signal-to-noise ratio at the array input and the desired accuracy of the DOA estimates. In the absence of noise, $N>d$ is required in order to completely span the signal subspaces. In the presence of noise, it has been shown that N must be at least m^2. Typically, if the SNR is known, N is chosen such that the Frobenius norm of the perturbations in $\hat{R}$ is 30 db below the covariance matrix norm.

Estimating d and σ^2

Due to errors in $\hat{R}_{xx}$, its eigenvalues will be perturbed from their true values and the true multiplicity of the minimal eigenvalue may not be evident. A popular approach for determining the underlying eigenvalue multiplicity is an information theoretic method based on the minimum description length (MDL) criterion. The estimate of the number of sources d is given by the value of k for which the following MDL function is minimized:

$$MDL(k) = -\log\left\{\frac{\prod_{i=k+1}^{m}\hat{\lambda}_i^{\frac{1}{m-k}}}{\frac{1}{m-k}\sum_{i=k+1}^{m}\hat{\lambda}_i}\right\}^{(m-k)N} + \frac{k}{2}(2m-k)\log N; \tag{18}$$

where $\hat{\lambda}_i$ are the eigenvalues of R_{xx}. The MDL criterion is known to yield asymptotically consistent estimates. Note that since R_{xx} and R_{xy} both span the same subspace (of dimension d), a method that efficiently exploits this underlying model will yield better results.

Having obtained an estimate of d, the maximum likelihood estimate of σ^2 conditioned on $\hat{d}$ is given by the average of the smallest $m-\hat{d}$ eigenvalues i.e.,

$$\hat{\sigma}^2 = \frac{1}{m-\hat{d}}\sum_{i=\hat{d}+1}^{m}\hat{\lambda}_i. \tag{19}$$

Estimating ASA* and ASΦ*A*

Using the results from the previous step, and making no assumptions about the array geometry, the maximum likelihood estimate $\hat{C}_{xx}$ of ASA*, conditioned on $\hat{d}$ and $\hat{\sigma}^2$, is the maximum Frobenius norm (F-norm) rank d approximation of $\hat{R}_{xx}-\hat{\sigma}^2 I$, i.e.,

$$\hat{C}_{xx} = \sum_{i=1}^{d}(\hat{\lambda}_i - \hat{\sigma}^2)\hat{e}_i\hat{e}_i^*; \tag{20}$$

where; $\{e_1, e_2, \ldots e_m\}$ are the eigenvectors corresponding to the ordered eigenvalues of $\hat{R}_{xx}$.

Similarly, given $\hat{R}_{xy}$ and $\hat{d}$, the maximum likelihood estimate ASΦ*A* is the maximum F-norm rank d approximation of $\hat{R}_{xy}$

$$AS\Phi^*A^* = \sum_{i=1}^{d}\hat{\lambda}_i^{xy}e_i^{xy}e_i^{xy*}; \tag{21}$$

where, $\{\hat{\lambda}_1^{xy}>\hat{\lambda}_2^{xy}> \ldots >\lambda_m^{xy}\}$ and $\{e_1^{xy}, e_2^{xy}, \ldots, e_m^{xy}\}$ are the eigenvalues and the corresponding eigenvectors of $\hat{R}_{xy}$.

As remarked earlier, the information in $\hat{R}_{xx}$ and $\hat{R}_{xy}$ can be jointly exploited to improve the estimates of the underlying subspace and therefore of the estimates of ASA* and ASΦ*A*. In situations where the array geometry (i.e., the manifold on which the columns of A lie) is known, these estimates can be further improved, but this is not pursued here since no knowledge of the array geometry is assumed.

Estimating Directions of Arrival

The estimates of the DOA's now follow by computing the the m GEV's of the matrix pair ASA* and ASΦ*A*. This is a singular generalized eigen-problem and needs more care than the regular case to obtain stable estimates of the GEV's. Note that since the subspaces spanned by the two matrix estimates cannot be expected to be identical, the $m-d$ noise GEV's will not be zero. Furthermore, the signal GEV's will not lie exactly on the unit circle. In practice, d GEV's will lie close to the unit circle and the remaining $m-d$ GEV's well inside and close to the origin. The d values near the unit circle are the desired estimates of Φ_{kk}. The argument of Φ_{kk} may now be used in conjunction with (16) to obtain estimates of the source directions. This concludes the detailed discussion of the algorithm.

Some Results

Estimation of the Number of Signals

In the algorithm detailed above, an estimate of the number of sources $\hat{d}$ is obtained as one of the first steps in the algorithm. This estimate is then used in subsequent steps as the rank of the approximations to covariance matrices. This approach has the disadvantage that an error (particularly underestimation) in determining d may result in severe biases in the final DOA estimates. Therefore, if an estimator for σ^2 can be found which is independent of d (e.g., $\hat{\sigma}^2=\hat{\lambda}_{min}$), estimation of d and the DOA's can be performed simultaneously. Simulation results have shown that the estimates of Φ have low sensitivity to errors in estimating σ^2. This implies that the rank d estimates of ASA* and ASΦ*A* can be dispensed with and the GEV's computed directly from the matrix pair $\{\hat{R}_{xx}-\hat{\sigma}^2 I, \hat{R}_{xy}\}$. This results in the need to classify the GEV's as either source or noise related which is a function of their proximity to the unit circle. This ability to simultaneously estimate d and the parameters of interest is another advantage of ESPRIT over MUSIC.

Extensions to Multiple Dimensions

The discussion hitherto has considered only single dimensional parameter estimation. Often, the signal parameterization is of higher dimension as in DF problems where azimuth, elevation, and temporal frequency must be estimated. In essence, to extend ESPRIT to estimate multidimensional parameter vectors, measurements must be made by arrays manifesting the the shift invariant structure in the appropriate dimension. For example, co-directional sensor doublets are used to estimate DOA's in a plane (e.g., azimuth) containing the doublet axes. Elevation angle is unobservable with such an array as a direct consequence of the rotational symmetry about the reference direction defined by the doublet axes (cf. cones of ambiguity). If both azimuth and elevation estimates are required, another pair of subarrays (preferably orthogonal to the first pair) sensitive to elevation angle is necessary. Geometrically, this provides an independent set of cones, and the intersections of the two sets of cones yield the desired estimates. Note that the parameter estimates (e.g., azimuth and elevation) can be calculated independently. This results in the computational load in ESPRIT growing linearly with the dimension of the signal parameter vector, whereas in MUSIC it increases exponentially.

If the signals impinging on the array are not monochromatic, but are composed of sums of cisoids of fixed frequencies, ESPRIT can also estimate the frequencies. This requires temporal (doublet) samples which can be obtained for example by adding a uniform tapped delay line ($p+1$ taps) behind each sensor. The frequencies estimates are obtained (independent of the DOA estimates) from the $mp \times mp$ auto- and cross-covariance matrices of two (temporally) displaced data sets (corresponding to subarrays in the spatial domain). The first set X contains mp samples obtained from taps 1 to p taps in each of the m delay lines behind the sensors. The set Y is a delayed version of X and uses taps 2 to $p+1$ in each of the m delay lines. The GE's obtained from these data sets define the multiple frequencies. Note that in time domain spectral estimation, ESPRIT is only applicable for estimating parameters of sums of (complex) exponentials. As mentioned previously, wideband signals can be handled by processing selected frequency components obtained via frequency selective narrowband (comb) filters.

Array Ambiguities

Array ambiguities are discussed below in the context of DOA estimation, but can be extended to other problems as well.

Ambiguities in ESPRIT arise from two sources. First, ESPRIT inherits the ambiguity structure of a single doublet, independent of the global geometry of the array. Any distribution of co-directional doublets contains a symmetry axis, the doublet axis. Even though the individual sensor elements may have directivity patterns which are functions of the angle in the other dimension (e.g., elevation), for a given elevation angle the directional response of each element in any doublet is the same, and the phase difference observed between the elements of any doublet depends only on the azimuthal DOA. The MUSIC algorithm, on the other hand, can (generally) determine azimuth and elevation without ambiguity given this geometry since knowledge of the directional sensitivities of the individual sensor elements is assumed.

Other doublet related ambiguities can also arise if the sensor spacing within the doublets is larger than $\lambda/2$. In this case, ambiguities are generated at angles arcsin $\{\lambda(\Phi_{ii} \pm 2n\pi)/2\pi\Delta\}$, $n=0, 1, \ldots,$ a manifestation of undersampling and the aliasing phenomenon.

ESPRIT is also heir to the subarray ambiguities usually classified in terms of first-order, second-order, and higher order ambiguities of the array manifold. For example, second-order, or rank 2 ambiguities occur when a linear combination of two elements from the array manifold also lies on the manifold, resulting in an inability to distinguish between the response due to two sources and a third source whose array response is a weighted sum of the responses of the first two. These ambiguities manifest themselves in the same manner as in MUSIC where they bring about a collapse of the signal subspace dimensionality.

Finally, it should be noted that the doublet related ambiguities present in ESPRIT do not cause any real difficulties in practice. Indeed, it is precisely such ambiguities that allow ESPRIT to separately solve the problem in each dimension.

Array Response Estimation and Signal Copy

There are parameters other than DOA's and temporal frequencies that are often of interest in array processing problems. Extensions of ESPRIT to provide such estimates are described below. ESPRIT can also be easily extended to solve the signal copy problem, a problem which is of particular interest in communications applications.

Estimation of Array Response (Direction) Vectors

Let e_i be the generalized eigenvector (GEV) corresponding to the generalized eigenvalue (GE) γ_i. By definition, e_i satisfies the relation

$$AS(I-\gamma_i\Phi)A^*e_i=0. \tag{22}$$

Since the column space of the pencil $AS(I-\gamma_i\Phi)A^*$ is the same as the subspace spanned by the vectors $\{a_j, j\neq i\}$, it follows that e_i is orthogonal to all direction vectors, except a_i. Assuming for now that the sources are uncorrelated, i.e.,

$$S=\mathrm{diag}(\sigma_1^2, \ldots, \sigma_d^2); \tag{23}$$

multiplying C_{xx} by e_i yields the desired result:

$$C_{xx}e_i=AS[0, \ldots, 0, a_i^*e_i, 0, \ldots, 0]^T=a_i(\sigma_i^2 a_i^* e_i)=\text{scalar}\times a_i. \tag{24}$$

The result can be normalized to make the response at sensor 1 equal to unity, yielding:

$$a_i=\frac{C_{xx}e_i}{u^T C_{xx} e_i}, \tag{25}$$

where $u=[1, 0, 0, \ldots, 0]^T$.

Estimation of Source Powers

Assuming that the estimated array response vectors have been normalized as described above (i.e., unity response at sensor 1), the source powers follow from (24):

$$\sigma_i^2=\frac{|u^T C_{xx} e_i|^2}{e_i^* C_{xx} e_i} \tag{26}$$

Note that these estimate are only valid if sensor 1 is omni-directional, i.e., has the same response to a given source in all directions. If this is not the case, the estimates will be in error.

Estimation of Array Geometry

The array geometry can now be found from $\{a_i\}$ by solving a set of linear equations. The minimum number of direction vectors needed is equal to the number of degrees of freedom in the sensor geometry. If more vectors are available, a least squares fit can be used. Note that multiple experiments are required in order to solve for the array geometry, since for each dimension in space about which array geometric information is required, m direction vectors are required. However, in order to obtain estimates of the direction vectors, no more than $m-1$ sources can be present during any one experiment. Thus the need for multiple experiments is manifest.

Signal Copy (SC)

Signal copy refers to the weighted combination of the sensor measurements such that the output contains the desired signal while completely rejecting the other $d-1$ signals. From (22), e_i is orthogonal to all wavefront direction vectors except the i^{th} wavefront, and is therefore the desired weight vector for signal copy of the i^{th} signal. Note that this is true even for correlated signals. If a unit response to the desired source is required, once again the assumption of a unit response at sensor 1 to this source becomes necessary. The weight vector is now a scaled version of e_i and using the constraint $a_i^* w_i^{SC}=1$ can be shown to be

$$w_i^{SC} = e_i\left(\frac{|u^T C_{xx} e_i|}{e_i^* C_{xx} e_i}\right). \tag{27}$$

In the presence of correlated signals as often arises in situations where multipath is present, it is useful to combine the information in the various wavefronts (paths). This leads to a maximum likelihood (ML) beamformer which is given by:

$$w_i^{ML} = R_{xx}^{-1} C_{xx} e_i. \tag{28}$$

In the absence of noise, $R_{xx}=C_{xx}$ and $w_i^{ML}=w_i^{SC}$. Similarly, optimum weight vectors for other types of beamformers can be determined.

Some Generalizations of the Measurement Model

Though the previous discussions have been restricted to specific models for the sensors elements and noise characteristics, ESPRIT can be generalized in a straightforward manner to handle a larger class of problems. In this section, more general models for the element, signal, and noise characteristics are discussed.

Correlated Noise

In the case when the additive noise is correlated (i.e., no longer equal to $\sigma^2 I$), modifications are necessary. If the noise auto- and cross-covariances for the X and Y subarrays are known to within a scalar, a solution to the problem is available. Let Q_{xx} and Q_{xy} be the normalized auto- and cross-covariance matrices of the additive noise at the subarrays X and Y. Then,

$$ASA^* = R_{xx} - \lambda_{min}^{(R_{xx},Q_{xx})} Q_{xx}; \tag{29}$$

where $\lambda_{min}^{(R_{xx},Q_{xx})}$ is the minimum GEV (multiplicity $m-d$) of the matrix pair (R_{xx}, Q_{xx}). We can also find

$$AS\Phi^*A^* = R_{xy} - \lambda_{min}^{(R_{xy},Q_{xy})} Q_{xy}, \tag{30}$$

where $\lambda_{min}^{(R_{xy},Q_{xy})}$ is similarly defined. At this point, the algorithm proceeds as before with the GE's of the matrix pair $(ASA^*, AS\Phi^*A^*)$ yielding the desired results.

Coherent Sources

The problem formulation discussed so far assumed that no two (or more) sources were fully correlated with each other. This was essential in the development of the algorithm to this point. ESPRIT relies on the property that the values of γ for which the pencil $(ASA^*-\gamma AS\Phi^*A^*)$ reduces in rank from d to $d-1$ determine Φ. This is, however, true only when

$$\rho(ASA^*-\gamma AS\Phi^*A^*)=\rho(S(I-\gamma\Phi))=\rho(I-\gamma\Phi). \tag{31}$$

That is, $\rho(I-\gamma\Phi)$ rather that $\rho(S)$ determines $\rho(ASA^*-\gamma AS\Phi^*A^*)$. This in turn is satisfied only when S is full rank, and thus excludes fully coherent sources.

ESPRIT can be generalized to handle this situation using the concept of spatial smoothing. Consider a signal environment where sources of degree two coherency (i.e., fully coherent groups contain at most two sources each) are present. Assume that the array is now made up of triplet (rather than doublets used earlier) element clusters. Let the corresponding subarrays be referred to as X, Y and Z. Assume, as before, that elements within a cluster are matched and all clusters have a identical (local) geometry. Let Φ_{XY} and Φ_{XZ} be the rotation operators with respect to subarray X for subarrays Y and Z respectively.

Defining the covariances R_{xx}, R_{yy}, R_{zz}, R_{xy}, and R_{xz} in the usual manner, we note that

$$C_{zz} = R_{zz} - \lambda_{min}^{zz} I = A\Phi_{XZ} S \Phi_{XZ}^* A^*. \tag{32}$$

and

$$R_{xz} = AS\Phi_{XZ}^* A^*,$$

$$R_{yz} = A\Phi_{XY} S \Phi_{XZ}^* A^*. \tag{33}$$

Now consider the matrix pencil

$$(C_{xx}+C_{zz})-\gamma(R_{xy}+R_{zy}) = A(S+\Phi_{XZ}S\Phi_{XZ}^*)(I-\gamma\Phi_{XY})A^*. \tag{34}$$

It is easy to show that for a degree two coherency model,

$$\rho(S+\Phi_{XZ}S\Phi_{XZ}^*)=d. \tag{35}$$

Therefore, the rank of the smoothed wavefront covariance matrix has been restored. Hence, $(I-\gamma\Phi)$ once again controls rank of the smoothed pencil in (34), and the GE's of the pair $\{C_{xx}+C_{zz}, R_{xy}+R_{zy}\}$ determine the DOA's. Further, for arbitrary degree of coherency it can be shown that the number of elements needed in a cluster is equal to the degree of coherency plus one.

Mismatched Doublets

The requirement for the doublets to be pairwise matched in gain and phase response (at least in the directions from which the wavefronts are expected) can be relaxed as shown below.

1. Uniform Mismatch—The requirement of pairwise matching of doublets can be relaxed to having the relative response of the sensors to be uniform (for any given direction) at all doublets. This relative response, however, can change with direction. Let A denote the direction matrix for subarray X. The the direction matrix for subarray Y can then be written as AG, where;

$$G = \mathrm{diag}(g_1, \ldots, g_d), \tag{36}$$

and $\{g_i\}$ are the relative responses for the doublet sensors in the directions θ_i. It is evident that the generalized eigenvalues of the matrix pair $\{C_{xx}, R_{xy}\}$ will now be $\Phi_{ii}G_{ii}$ resulting in GE's which no longer lie on the unit circle. If the relative gain response (G_{ii}) is real, the GE's deviate only radially from the unit circle. Since it is the argument (phase angle) of the GE's which is related to the DOA's, this radial deviation is important only in so far as the method of determining the number of signals must be altered (the number of unit circle GE's is no longer $\hat{d}$). On the other hand, a relative phase response will rotate the GE's as well resulting in estimation bias that can be eliminated only if the relative phase mismatch is known. As an example of such an array of mismatched doublets, consider X and Y subarrays which are identical across each subarray but are mismatched between arrays.

2. Random Gain and Phase Errors—In practice, sensor gains and phases may not be known exactly and pairwise doublet matching may be in error violating the model assumptions in ESPRIT. However, techniques are available that exploit the underlying signal model to identify the sensor gains and phase from the sensor data. This is in effect a pseudo-calibration of the array where data from a few experiments are used to identify gain and phase error parameters. The estimates so obtained are the used to calibrate the doublets.

A Generalized SVD Approach

The details of the computations in ESPRIT presented in the previous sections have been based upon the estimation of the auto- and cross-covariances of the subarray sensor data. However, since the basic step in the algorithm requires determining the GE's of a singular matrix pair, it is preferable to avoid using covariance matrices, choosing instead to operate directly on the data. Benefits accrue not only from the resulting reduction in matrix condition numbers, but also in the potential for a recursive formulation of the solution (as opposed to the block-recursive nature of eigendecomposition of sample covariance matrices). This approach leads to a generalized singular value decomposition (GSVD) of data matrices and is briefly described below.

Let X and Y be $m \times N$ data matrices containing N simultaneous snapshots $x(t)$ and $y(t)$ respectively;

$$X = [x(t_1), x(t_2), \ldots, x(t_N)],$$

$$Y = [y(t_1), y(t_2), \ldots, y(t_N)]. \quad (37)$$

The GSVD of the matrix pair (X, Y) is given by:

$$X = U_X \Sigma_X V^*,$$

$$Y = U_Y \Sigma_Y V^*, \quad (38)$$

where U_X and U_Y are the $m \times m$ unitary matrices containing the left generalized singular vectors (LGSV's), Σ_X and Σ_Y are $m \times N$ real rectangular matrices that have zero entries everywhere except on the main diagonal (whose pairwise ratios are the generalized singular values), and V is a nonsingular matrix.

Assuming for a moment that there is no additive noise, both X and Y will be rank d. Now consider the pencil

$$X - \gamma Y = A(I - \gamma\Phi)[s(t_1), \ldots, s(t_N)]. \quad (39)$$

Similar to previous discussions, whenever $\gamma = \Phi_{ii}$, this pencil will decrease in rank from d to $d-1$. Now consider the same pencil written in terms of its GSVD:

$$\begin{aligned} X - \gamma Y &= (U_X\Sigma_X - \gamma U_Y\Sigma_Y)V^* \\ &= U_X\Sigma_X(I - \gamma\Sigma_X^{-1}U_X^* U_Y\Sigma_Y)V^* \end{aligned} \quad (40)$$

This pencil will loose rank whenever γ is an eigenvalue of $(\Sigma_X^{-1}U_X^*U_Y\Sigma_Y)$. Therefore the desired Φ_{ii} are the eigenvalues of the product $\Sigma_X^{-1}U_X^*U_Y\Sigma_Y$. However, from the underlying model in (1) and (2), it can be shown that in the absence of noise $\Sigma_X = \Sigma_Y$, in which case Φ_{ii} are also the eigenvalues of $U_X^*U_Y$.

In presence of additive white sensor noise, we can show that asymptotically (i.e., for large N) the GSVD of the data matrices converges to the GSVD obtained in the noiseless case except that Σ_X and Σ_Y are augmented by $\sigma^2 I$. Therefore, the LGSV matrices in the presence of noise are asymptotically equal to U_X and U_Y computed in the absence of noise, and the earlier result is still applicable.

To summarize, when given data instead of covariance matrices, ESPRIT can operate directly on the data by first forming the data matrices X and Y from the array measurements. Then, the two LGSV matrices U_X and U_Y are computed. The desired Φ_{ii} are then computed as the eigenvalues of the product $U_X^*U_Y$. Estimates for other model parameters as discussed previously can be computed in a similar manner.

What is claimed is:

1. A method of locating signal sources and estimating source parameters comprising the following steps:
   (a) providing an array of groups of signal sensor pairs, the sensors in each pair in each group being identical except for a fixed displacement which may differ from group to group, thereby defining two subarrays (X and Y) in each group,
   (b) obtaining signal measurements with the sensor array so configured,
   (c) determining from said signal measurements the auto-covariance matrix R_{xx} of the X subarray in each group and the cross-covariance matrix R_{xy} between the X and Y subarrays in each group,
   (d) determining the smallest eigenvalue of the covariance matrix,
   (e) subtracting said smallest eigenvalue from each element of the principal diagonal of the covariance matrix R_{xx} and obtaining a difference C_{xx},
   (f) determining the generalized eigenvalues of the matrix pair (C_{xx}, R_{xy}), and
   (g) locating the generalized eigenvalues which lie on a unit circle, the number of which corresponding to the number of sources and the locations of which corresponding to the parameter estimates.

2. The method as defined by claim 1 and further including the steps of:
   (a) varifying specific signal reception by determining array response (direction) vectors using the generalized eigenvectors, and
   (b) estimating the array geometry from the said array response vectors.

3. The method as defined in claim 1 with variations to improve numerical characteristics using generalized singular value decompositions of data matrices instead of generalized eigendecomposition of covariance matrices by:
   (a) forming data matrices X and Y from the data from the subarrays in each group,
   (b) computing the generalized singular vectors of the matrix pair (X,Y) yielding $X = U_X\Sigma_X V^*$ and $Y = U_Y\Sigma_Y V^*$,
   (c) computing the eigenvalues of $\Sigma_X^{-1}U_X^*U_Y\Sigma_Y$ and
   (d) locating those eigenvalues which lie on or near the unit circle, the number of which corresponding to the number of sources and the locations of which corresponding to the parameter estimates.

* * * * *

Exhibit F

EXHIBIT F

ArrayComm, Inc.

Corporate Overview

17 December 1993

ArrayComm, Inc.
3255 Scott Blvd.
Bldg. 4, Suite 103
Santa Clara, CA 95054

TEL: (408) 982-9090
FAX: (408) 982-9092

1. Introduction

The wireless communications industry is exploding. The intelligent antennas created by ArrayComm, Inc. add a new dimension to the technologies that will provide the foundation for personal communication in the 21st century.

In mobile communications applications, the technology has demonstrated its ability to substantially reduce cost by decreasing the infrastructure needs of new systems. In addition, the subscriber capacity of existing and new systems is increased and interference reduced. The technology can also benefit other high-growth fields such as wireless local-loops, point-to-point distribution systems, transportation systems, and space systems.

Cellular communications is already a $25 billion/year industry worldwide and is the fastest growth area in the electronics field. The industry is expected to grow to $100 billion annually by the year 2000.

The advantages and improvements offered by ArrayComm can accelerate this growth, especially in emerging economic areas of the world that do not have sufficient wired network capacity. Major US corporations are currently entering into agreements with the governments of Third World countries and with Eastern European nations to install wireless cellular systems as the main telecommunications network throughout their respective countries. With a finite number of service providers worldwide, ArrayComm's technology is expected to be adopted quickly.

2. Historical Background

From the later part of the 1970s through the mid 1980s, Dr. Richard Roy, as part of a team at Stanford University, developed the mathematical underpinnings of the technology he later named SDMA. Meanwhile, over the last decade, the wireless communication market has developed and grown to such a point that the need for the new technology has become acute. In order to meet the need, Dr. Roy assembled a management team of high-quality telecommunications executives, internationally known communications marketing professionals, and expert legal and financial counsel, and ArrayComm was formed in April 1992.

3. Business Areas

As mentioned, the principal fields and applications for SDMA technology include but are not limited to wireless telecommunications networks such as:

- Personal communication services (PCS)
- Cellular mobile communication systems
- Wireless local loop
- Acknowledgement paging systems
- Air-to-ground (airphone) communication systems

- Special Mobile Radio (SMR)
- Private Land Mobile Radio (PLMR)
- Wireless local area computer networks
- Personal digital assistants communication systems
- Satellite communication systems

While ArrayComm's SDMA is a genuine, proven technological breakthrough with important implications in each of these areas, the most immediate application allowing the largest commercial potential relates to communication systems such as cellular telephone and personal communication systems.

Current telecommunication systems contain inherent limitations with regard to capacity. As more and more users join the system, the frequencies simply become crowded. Wireless units transmitting on the same channel cannot be resolved by the receiver since there is no way of distinguishing signals that share the same frequency. The result to the end user is dropped calls, poor reception, interference (cross-talk) and noise.

SDMA effectively combats these problems. By implementing the new technology, telecommunications systems realize substantial increases in capacity, and moreover, quality is also greatly improved. Consequently, the mobile unit transmitted power can be reduced, resulting in longer battery life.

As noted, **the technology is compatible with current technologies, both digital and analog, and with equipment now in use.** In addition, implementation can occur on a cell by cell basis, where and as needed, and with a relatively low capital cost since no exotic hardware is required.

The technology is *also* suited to new wireless system deployment. The flexibility afforded to system designers is advantageous, and the resultant cost benefits are substantial. Preliminary calculations for deployment of PCS systems utilizing the technology, for example, indicate a cost-savings on the order of 50%. Similar savings are projected for new wireless local-loop, paging and air-to-ground services to be deployed over the next decade.

The technology offered by ArrayComm is protected by two current US patents while two others are pending.

4. Management and Operations

Located at the heart of Silicon Valley in Santa Clara, CA, ArrayComm has ready access to a large pool of technical and manpower resources. ArrayComm's engineering team is led by Drs. Roy and Barratt, and includes experts in various areas of signal processing technology. ArrayComm's management team is led by Martin Cooper, who with 35 years in the field is one of the best known personalities in the industry.

ArrayComm is forming a European subsidiary, ArrayComm Europa, which will be headed by Mr. Maurice Remy, who most recently headed Matra Communications, a large European telecommunications company. ArrayComm Europa will be responsible for pursuing the various opportunities afforded by the technology in Europe.

The credentials and international recognition of its technical and management teams coupled with the unique benefits of its technology places ArrayComm at the cutting edge of telecommunications and signal processing technology. This will allow it to continue to establish joint-ventures or strategic alliances with major manufacturing and operating firms.

5. Accomplishments and Outlook

During the past eighteen months, ArrayComm has:

- developed a wide range of domestic and international contacts with telecommunication equipment manufacturers and service operators. These contacts have already led to written statements of interests from such European and American manufacturers and from several major cellular providers.
- completed a proof-of-concept demonstration that illustrates the capabilities of the IntelliCell intelligent antenna based on SDMA.
- filed two patent applications in addition to the basic patents to which it has exclusive rights.
- financed the above through private groups rather than from industry sources, in order to maintain its independence.

ArrayComm's basic business strategy includes the protection of its capital resources through a strategy of joint ventures, licensing, and co-development. One or more of these relationships is expected to be established in the coming months.

For more information on the future of ArrayComm, contact Arnaud Saffari at (408) 982-9080.

ArrayComm, Inc.'s Management

ArrayComm consists of an experienced and high-powered management group, headed by Martin Cooper one of the pioneers of radio-telephony in the United States, combined with a top-flight technical team headed by Dr. Richard Roy, the primary inventor of SDMA. The Company has recently bolstered its management and technical capabilities by enlisting the assistance of several experienced outside directors and the members of its Technical Advisory Board. The Company strives to ensure a smooth and controlled decision-making process and to allow the technical team the ability to concentrate its efforts on the primary tasks of developing the Company's technology and providing service to its partners and clients.

Beside a Board of Directors, including five outside directors, and the operational sections in Santa Clara, various units have been formed to assist the company in its strategic development:

- A Technical Advisory Board includes luminaries from industry and academia such as Dr. William J. Perry, Secretary of Defense (on leave), and Professor Stephen Boyd of Stanford University.

- A European unit, which will be the core of the future European subsidiary, led by Mr. Maurice Remy, member of the Board of Directors and Mr. Georges Kasparian.

Management Biographies

Martin Cooper, Chairman of the Board and CEO Martin Cooper is also chairman of Spatial Communications, Inc., Cellular Pay Phone, Inc., and Dyna, Inc. and serves on the boards of several other companies. He is widely recognized as a pioneer in the personal communications industry and as an innovator in the management of research and development. He is an inventor who introduced in 1973 the first portable cellular radiotelephone, and is widely regarded as the father of cellular telephony. Mr. Cooper has wide industry experience including both in large corporate settings, and in successful entrepreneurial "start-up" contexts. Mr. Cooper founded and managed Cellular Business Systems, Inc. (CBSI), growing it to become the industry leader in cellular billing with a market share of approximately 75%, and selling the company to Cincinnati Bell. (Before its acquisition, CBSI provided billing and management services to most cellular companies in the U.S.)

Before that, he was Corporate Director of Research and Development for Motorola, Inc., responsible for the creation and stimulation of technology throughout Motorola. He joined Motorola in 1954 as a research engineer and advanced through a number of engineering and management positions before becoming a corporate officer in 1969 and vice president and general manager of the Communications Systems Division in 1977. During his 29 years at Motorola, Mr. Cooper oversaw the creation of a number of major businesses including high-capacity paging with annual sales in 1990 over $600 million, trunked mobile radio systems (known as SMRS) with annual sales over $1 billion, and cellular radio telephony with annual sales over $1 billion. Products introduced by Mr. Cooper have had cumulative sales volume of over $7 billion. While at Motorola, he had top secret clearances while participating in and managing highly classified government development programs. He advised the Motorola Foundation in its charitable endowments and contributions, managed its central research laboratories, and was Motorola's technology leader.

Mr. Cooper has been involved in industry and government efforts to allocate new radio frequency spectrum for the land mobile radio services and has been granted six patents in the communications field. He has been widely published on various aspects of communications technology and on management of research and development. Mr. Cooper is a graduate of the Illinois Institute of Technology with bachelors and masters degrees in electrical engineering. He is a Fellow of the Institute of Electrical and Electronic Engineers and of the Radio Club of America and is a member of ETA Kappa Nu (electrical engineering honorary) and Rho Epsilon (radio engineering honorary). He served in various offices of the Vehicular Technology Society of the IEEE and was president of the society in 1972 and 1973. Mr. Cooper was awarded the IEEE Centennial Medal in 1984. Mr. Cooper has served on technical committees of the Electronic Industries Association and the National Research Council as well as numerous industry and civic groups. He is a Distinguished Lecturer for the National Electronics Consortium and serves on its Board of Directors.

Richard H. Roy, President, Director Dr. Roy is the lead inventor of SDMA technology. Dr. Roy has been associated with Stanford University since 1972 and was granted an MSEE and a Ph.D. from that school. Prior to this he was granted a BS in Physics and Electrical Engineering from the Massachusetts Institute of Technology. His professional experience includes [1985--1987] research scientist for Integrated Systems, Inc., [1983--1985] research scientist with MacLeod Laboratories, Inc., [1975--1984] senior member of the technical staff of ESL, Inc.. involved in the development of state-of-the-art techniques in estimation, identification, real-time signal processing and information extraction and adaptive control for various aerospace applications. His fields of research have focused on multidimensional signal parameter estimation, signal processing theory, and adaptive algorithms. He is widely published internationally, has been invited to speak at conferences around the world, and has been granted two patents in connection with the development of SDMA.

Arnaud Saffari, Vice President Marketing and COO Mr. Saffari was granted an MSEE from Ecole Superieure d'Electricite de Paris, France. Since spending six years setting-up and running a major broadcasting network in the Middle-East, Mr. Saffari has been an independent international marketing consultant for the past twenty years. He has successfully organized the development, award and management of major international projects and contracts ranging from $15 million to $85 million for U.S. and European clients. His U.S. clients have included General Electric Space Division, Westinghouse Government Systems Division, Granger Associates, Arthur D. Little, and Telemation. Among his past and present European clients are Thomson-CSF, CIT-Alcatel, Alcatel Espace, Robert Bosch Gmbh, Rohde and Schwarz, CGTI, Drusch, Cremer, Camusat S.A., Rank, and EMI. Mr. Saffari has successfully managed various large high-tech businesses with staff under him as large as 1100 persons, and has had bottom line responsibility for these organizations. Mr. Saffari has authored several publications on Forecasting of Communications Technology and Price Modeling.

Craig Barratt, Vice President, Engineering Dr. Barratt was granted MSEE and Ph.D. degrees from Stanford University. Prior to these, he was granted a BE (Honors) in Electrical Engineering and a BS in Pure Mathematics and Physics from Sydney University (Australia). He has extensive R&D experiance in electronics, computer systems hardware and software, and signal processing, acquired with several Silicon Valley firms including Resonex, Inc., where he had been System Architect for a whole body magnetic resonance imaging (MRI) machine. From 1978 until moving to the U.S. in 1984, Dr. Barratt also had extensive experience as a hardware engineer for several Australian firms where he designed, prototyped, and managed through to production several computer systems and peripherals. Dr. Barratt has also taught graduate Electrical Engineering courses at Stanford University.

Maurice Remy, Director Mr. Maurice Remy is a graduate of France's prestigious Ecole Polytechnique. After a long career in the technical services of France's Radio and TV broadcasting networks (ORTF), Mr. Remy headed for several years the Central Research Laboratories of ORTF, then was appointed Chairman and CEO of Telediffusion de France, the integrated French television transmission organization serving all networks. In 1983, Mr. Remy was recruited by Matra Group, one of France's foremost technology groups, to head its fledgling telecommunications company, Matra Communications. Over the span of nine years at Matra, until his retirement in late 1992, Mr. Remy built Matra Communications into one of the best known and strongest telephone and cellular systems manufacturers in Europe with sales of $1.3 billion.

Georges Kasparian, VP Marketing Europe Mr. Kasparian was granted an MSEE from Ecole Superieure d'Electricite de Paris, France. Mr. Kasparian has spent over 30 years working for major European electronics firms in senior sales and marketing positions. He was in succession sales manager for government electronics at Philips, Deputy Director of Sales for Thomson-CSF Broadcast Equipment Division, with worldwide responsibilities, Vice President of International Marketing and Sales for Matra International Division, and Regional Vice-President for Alcatel Trade International. Since 1988, Mr. Kasparian has very successfully pursued an independent practice as an International Marketing Consultant on Electronic Systems for major European companies.

Mario M. Rosati, Esq., Director, General Counsel Mr. Rosati is a member in the Palo Alto, California law firm of Wilson, Sonsini, Goodrich and Rosati and has been with the firm since 1971. Mr. Rosati received his law degree from Bolt Hall, University of California, Berkeley. Mr. Rosati specializes in corporate law, especially as it relates to high technology companies. He is a director of the following California firms: Genus, Inc., Aehr Test Systems, Pro-Log Corporation, CATS Software, Inc., and Tulip Memory Systems. In addition, he is counsel for a number of corporations including Sierra Semiconductor, Genpharm International, Inc., Menlo Care, Inc., Vivus, Inc., Ross Systems, Viewstar Corporation, and Cell Genesys.

Exhibit G

EXHIBIT G

TM SRA-NSF-92-1

Increasing Capacity in Wireless Information Networks

SDMA - A New Concept in Mobile Communications

May 9, 1993

SBIR Phase I - Final Report

Dr. R. H. Roy
Systems Research Associates
10030 Phar Lap Drive
Cupertino, CA 95014

Prepared for: Data Support Services
National Science Foundation
1800 G Street, N.W., Room 223
Washington, D.C. 20550
ATTN: SBIR

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1. Introduction

Wireless communications networks (WCN's) are an increasingly pervasive mechanism for the interchange of data in the United States and throughout the rest of the world. As the radio spectrum available to these systems is limited, efficient spectral utilization is required to satisfy the growing demand for their services. The capacity of some systems has already been exhausted, for example cellular telephone systems operating in certain urban areas. Several signaling or modulation schemes have been proposed to increase the spectral efficiency of these systems: Code Division Multiple Access (CDMA), Frequency Division Multiple Access (FDMA), and Time Division Multiple Access (TDMA) provide representative examples. Although efficient modulation is an important component of a well-designed WCN, it fails to alleviate what is perhaps the most important and common spectral inefficiency present in commercial WCN's — spatial inefficiency.

Most WCN's provide point-to-point (*e.g.* base station to user, or user to user) links rather than broadcast (*i.e.* base station to all users) links. Yet these same systems typically broadcast radio frequency energy omnidirectionally or, at best, with only a small measure of directivity. Selective directional transmission and reception increases the capacity of these systems by supporting multiple links to separated users on the same frequency, at the same time lowering the transmitted power requirements for the base station and for the users' units. It improves the signal quality of these systems through the elimination of co-channel interference or crosstalk. This approach, utilizing arrays of antennas and sophisticated digital signal processing techniques, is referred to as Spatial Division Multiple Access or SDMA. SDMA is compatible with all current or proposed modulation schemes for WCN's; and it can be incorporated into an existing base station while retaining full compatibility with the users' existing equipment.

Algorithmically, SDMA is rooted in a collection of signal processing algorithms developed at Stanford University and elsewhere during the late 1970's and 1980's. These algorithms are collectively referred to as *subspace-based estimation and detection algorithms* and have been successfully applied in a variety of applications: time series analysis, system identification and, now, WCN's. In WCN applications, subspace-based algorithms make it possible to separate multiple signals operating in the same frequency band from each other, from background noise or interference, and from propagation effects such as multipath. Details of SDMA's algorithmic components are provided in Appendix A.

Figure 1-1 illustrates a typical WCN base station employing SDMA. To place our example in a specific context, we will assume that the WCN is an Advanced Mobile Phone Service (AMPS) cellular telephone system. In AMPS systems, users transmit to the base station on one frequency and receive on a different frequency, these frequencies are respectively denoted by f_{c1} and f_{c1} in the figure. The AMPS specification provides for a single connection per receive-transmit frequency pair per cell. With SDMA, multiple conversations per frequency pair are possible. The right side of the figure depicts the processing of received signals, the processing of transmitted signals is depicted on the left. Shaded boxes indicate SDMA components, unshaded boxes indicate the components of a conventional AMPS base station.